Hot X: Algebra Exposed

Solution Guide for Chapter 18

Here are the solutions for the "Doing the Math" exercises in *Hot X: Algebra Exposed*!

DTM from p.261-262

2. What do we do with a negative exponent? We say, "Hey, you're on the wrong floor!" Then we simplify. So, $\frac{1}{2^{-4}} = \frac{2^4}{1} = 2^4 = \mathbf{16}$.

Answer: 16

3. Since this is written as a product of factors, we can go ahead and flip the bases with negative exponents, and we get: $\frac{(e^0 + f^0)f^{-1}}{e^{-3}} = \frac{(e^0 + f^0)e^3}{f^1}$ and notice that the zero exponents neutralize their victims, so we end up with: $\frac{(e^0 + f^0)e^3}{f^1} = \frac{(1+1)e^3}{f} = \boldsymbol{\frac{2e^3}{f}}$

Answer: $\boldsymbol{\frac{2e^3}{f}}$

4. We can't distribute exponents over subtraction, so let's simplify the inside by allowing those zero exponents to neutralize everything they touch, and we get:

$x\left[3x^0 - (3x)^0\right]^2 = x\left[3(1) - (1)\right]^2 = x\left[3-1\right]^2 = x\left[2\right]^2$

Ah, much better! And we get $x(4) = \mathbf{4x}$

Answer: 4*x*

5. First we'll distribute those outside exponents, and on the first one, the entire thing inside the parentheses becomes 0! And distributing the second parentheses' exponent, our problem becomes:

$$1 + 5^{-2} \cdot x^{0\cdot(-2)} \cdot z^{-2} \quad = \quad 1 + 5^{-2} \cdot x^{0} \cdot z^{-2} \quad = \quad 1 + 5^{-2} \cdot 1 \cdot z^{-2} \quad = \quad 1 + 5^{-2} \cdot z^{-2}$$

Now, dealing with those negative exponents, we get: $1+\frac{1}{5^2 \cdot z^2} \; = \; \mathbf{1+\frac{1}{25z^2}}$

Answer: $\mathbf{1+\frac{1}{25z^2}}$

6. In this case, we can simply flip the positions of the bases – they're both on the wrong floor! So: $\frac{2^{-3}}{a^{-2}} \; = \; \frac{a^2}{2^3} \; = \; \mathbf{\frac{a^2}{8}}$

Answer: $\mathbf{\frac{a^2}{8}}$

7. The key is to look at that parentheses as a single unit, and not worry about what's inside it just yet. Then we can see how our first step can be to simply flip its position in the fraction: $\frac{1}{(c^{-3}+d^0)^{-1}} \; = \; \frac{(c^{-3}+d^0)^1}{1} \; = \left(c^{-3}+d^0\right) = \; c^{-3}+d^0$

Much better looking! And now we can deal with the negative and zero exponent, and we get: $\mathbf{\frac{1}{c^3}+1}$

Answer: $\mathbf{\frac{1}{c^3}+1}$

DTM from p.267-268

2. Let's deal with the outermost negative exponent first, and we get: $\dfrac{1}{(6^1)(7^{-1})}$. Next, we'll flip 7's position in the fraction and lose the invisible exponents, and we get:

$\dfrac{7^1}{6^1} = \mathbf{\dfrac{7}{6}}$

Answer: $\mathbf{\dfrac{7}{6}}$

3. Getting rid of the negative exponents, we get: $\dfrac{32 \cdot 2}{b^6 \cdot b^5}$, and then using the product rule on the bottom and multiplying things, we get: $\boldsymbol{\dfrac{64}{b^{11}}}$. Done!

Answer: $\boldsymbol{\dfrac{64}{b^{11}}}$

4. Best to do these one step at a time and not get confused by all those negatives! First let's flip the whole thing upside down, getting rid of that outermost negative exponent, and we get: $\dfrac{2^{-4}}{(-4)^{-2}}$.

Next, we notice that everyone's on the wrong floor, so (still not worrying about the negative sign on the 4), we can do this: $\dfrac{2^{-4}}{(-4)^{-2}} = \dfrac{(-4)^2}{2^4}$

Finally, we're ready to actually apply those exponents to their bases, and get:

$\dfrac{(-4)^2}{2^4} = \dfrac{16}{16} = \mathbf{1}$. How about that!

Note: we also could have started by distributing the outside exponent to the inside, and we'd have gotten: $\left(\dfrac{(-4)^{-2}}{2^{-4}}\right)^{-1} = \dfrac{(-4)^{-2\times(-1)}}{2^{-4\times(-1)}} = \dfrac{(-4)^2}{2^4}$ and it would have gotten us the same answer!

Answer: 1

5. Let's take the hint and start by flipping the whole thing by applying the outside exponent, and we get: $\frac{w^7(1-y)}{w^3(2+x)}$. Next we can apply the quotient rule and subtract *w*'s exponents: 7 – 3 = 4, so we end up with a "4" exponent up top, and we get: $\mathbf{\frac{w^4(1-y)}{(2+x)}}$.

The parentheses on the bottom no longer serve a purpose, so we can drop them. And that's all we can do!

Answer: $\mathbf{\frac{w^4(1-y)}{2+x}}$

6. Remember fraction division? We just flip the second fraction and change division into multiplication, and in this case, we get: $\left(\frac{3q^{-1}}{3^{-1}}\right)^{-2} \times \frac{q}{1}$. We must deal with that "–2" exponent before we can proceed with the multiplication. So, distributing it, we get: $\frac{3^{-2}q^2}{3^2} \times \frac{q}{1}$. Let's put that top "3" where it belongs: $\frac{q^2}{3^2 \cdot 3^2} \times \frac{q}{1}$. And now we finish simplifying: $\frac{q^2}{3^2 \cdot 3^2} \times \frac{q}{1} \quad = \quad \frac{q^2 \cdot q}{3^2 \cdot 3^2 \cdot 1} \quad = \quad \mathbf{\frac{q^3}{81}}$. Done!

Answer: $\mathbf{\frac{q^3}{81}}$